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PROBLEMS OF CONSTRUCTING AN EFFICIENT COMPUTATIONAL ALGORITHM FOR RESTORING THE PARAMETERS OF THE STURM-LOUISVILLE OPERATOR

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KEYWORDS

Sturm-Liouville operator, eigenvalue, eigenfunction, calculation accuracy, algorithm, Gelfand-Levitan integral equation, integral sum

ABSTRACT

In this article, according to a given sequence of eigenvalues $\{\lambda_k\}_0^N$, $\{\mu_k\}_0^N$ with an accuracy of $O\left(\frac{1}{N^6}\right)$, the solution of the inverse problem for the Sturm-Liouville operator with an accuracy of $O\left(\frac{1}{N^6}\right)$ is found. The issue of constructing an effective calculation algorithm is discussed.

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SHTURM-LIUVILL OPERATORI PARAMETRLARINI TIKLASHDA SAMARALI HISOBLASH ALGORITMINI QURISH MASALALARI

KALIT SO‘ZLAR/
КЛЮЧЕВЫЕ СЛОВА:
Shturm-Liuvill operatori, xos qiymat, xos funksiya, hisoblash aniqligi, algoritm, Gelfand-Levitan integral tenglamasi, integral yig’indi

ANNOTATSIYA/АННОТАЦИЯ

Ushbu maqolada $O\left(\frac{1}{N^6}\right)$ aniqlikda berilgan $\{\lambda_k\}_0^N$, $\{\mu_k\}_0^N$ xos qiymatlar ketma-ketligi yordamida Shturm- Liuvill operatori uchun teskari masalaning $O\left(\frac{1}{N^6}\right)$ aniqlikdagi yechimini topish masalasi muhokama qilinadi.

Kirish. Bizga ko’pgina masalalarni yechida asosiy vosita bo’lib xizmat qiladigan quyidagi

$$y'' - [\lambda - q(x)]y = 0 \tag{1}$$

$$\begin{cases} y'(0) - hy(0) = 0 \\ y'(\pi) + Hy(\pi) = 0 \end{cases} \tag{2}$$

$$\begin{cases} y'(0) - h_1 y(0) = 0 \\ y'(\pi) + Hy(\pi) = 0 \end{cases} \tag{3}$$

chegaraviy masalalar berilgan bo‘lsin[1].

Agar (1)-(2) va (1)-(3) chegaraviy masalalarga mos keluvchi $\lambda_1, \lambda_2, \ldots, \lambda_n, \ldots$ va $\mu_1, \mu_2, \ldots, \mu_n, \ldots$ xos qiymatlar seriyasi berilgan bo‘lsa, (1)-(2) tenglamalarda qatnashayotgan *h, H, q(x)* parametrlarni to‘liq aniqlash mumkin[1]. Nazariy jihatdan yaxshi o’rganilgan ushbu masala

$$K(x,y) + F(x,y) + \int_0^x K(x,s)F(s,y)ds = 0 \tag{4}$$

ko‘rinishidagi Gelfand-Levitan integral tenglamasini yechishga keltiriladi. Bu yerda

$$F(x,y) = \sum_{k=0}^{\infty} \left[\frac{\cos(\lambda_k x)\cos(\lambda_k y)}{\alpha_k} - \frac{2\cos(kx)cos(ky)}{\pi}\right] \tag{5}$$

$$K(x,x) = h + \frac{1}{2}\int_0^x q(t)dt,$$

$$h = K(0,0) = -F(0,0) \tag{6}$$

$$\alpha_n = \frac{h_2 - \mathrm{h}_1}{\mu_n - \lambda_n} \prod_{\substack{k=1 \\ k\neq n}}^{\infty} \frac{\lambda_k - \lambda_n}{\mu_k - \lambda_n} \tag{7}$$

Ammo, hisoblash matematikasi nuqtai nazaridan qaralganda masalani Gelfand - Levitan sxemasi bo’yicha hisoblashda bir qancha savollarga aniqlik kiritish talab qilinadi[2], [3].

Ushbu maqolada chekli sondagi $\lambda_1, \lambda_2, \ldots, \lambda_N$ va $\mu_1, \mu_2, \ldots, \mu_N$ xos qiymatlar ketma-ketligi $O(N^{-6})$ xatolik bilan berilgan holatda (1)-(2) Shturm-Liuvill operatori paramerlarni

$O(N^{-6})$ aniqlikda topish masalasi muhokama qilinadi. Qaralayotgan masala o'z xususiyatiga ko'ra ma'lumotlar etarlicha berilmagan sharoitda $q(x)$ potensialni tiklash bilan bog'liq nokorrekt masaladir.

Maqolada klassik usuldan chetlashgan holda noma'lum $q(x)$ ni topish uchun Gelfand-Levitan integral tenglamasi o'rniga

$$\alpha_k = \int_0^{\pi} [\mathrm{y}(t, \lambda_k)]^2 dt \tag{7'}$$

tenglikdan foydalanishni tavsiya qiladi. Bu bilan (5) cheksiz trigonometric qatorni [0, π]×[0,π] kvadratning har bir bo'linish nuqtasida hisoblash va (4) integral tenglamasini echish bilan bog'liq muammolar bartaraf qilinadi. Shuningdek cheksizlik qatnashayotgan (6) va (7) formulalarni $O(N^{-6})$ aniqlikda hisoblash mumkin bo'lgan chekli algebraik ifoda bilan almashtirish, $(7')$ tenglikdan $q(x)$ potensialni anglash mumkin bo'lgan chiziqli tenglamalar sistemasini qurish masalasi muhokama qilinadi. Qo'yilgan masalani echishda Gelmfand-Levitan integral tenglamasi o'rniga $(7')$ tenglamadan foydalanish fikri maqola muallifi tomonidan taklif qilingan bo'lib, qator ilmiy amaliy masalalarni yechshda ushbu usuldan foydalanilgan [4]-[8].

Asosiy qism. Qo'yilgan masalani to'liq yechish bir necha bosqichdan iborat.

1. Ushbu bosqichda (1)-(2), (1)-(3) masalalarning spectral ma'lumotlari asimptotikalari koeffisientlari quyida keltirilgan lemma asosida yetarli aniqlikda hisoblab topiladi.

Lemma. Agar (1)-(2) va (1)-(3) chegaraviy masalalarga mos keluvchi $\lambda_1, \lambda_2, \ldots, \lambda_n, \ldots$ va $\mu_1, \mu_2, \ldots, \mu_n, \ldots$ xos qiymatlar ketma ketligining daslabki N ta hadlari $O(N^{-6})$ aniqlikda berilgan bo'lsa yetarli katta n>N lar uchun o'rinli bo'ladigan

$$\lambda_n = n^2 + a_0 + \frac{a_1}{n^2} + \frac{a_2}{n^4} + \cdots \tag{8}$$

$$\mu_n = n^2 + b_0 + \frac{b_1}{n^2} + \frac{b_2}{n^4} + \cdots \tag{8'}$$

$$s_n = \sqrt{\lambda_n} = n + \frac{c_0}{n} + \frac{c_1}{n^3} + \frac{c_2}{n^5} + \cdots \tag{9}$$

$$\sqrt{\mu_n} = n + \frac{c'_0}{n} + \frac{c'_1}{n^3} + \frac{c'_2}{n^5} + \cdots \tag{9'}$$

$$\alpha_n = \frac{\pi}{2} + \frac{a'_0}{n^2} + \frac{a'_1}{n^4} + \frac{a'_2}{n^6} + \cdots \tag{10}$$

asimptotikalarning koeffisientlarni quyidagi

$$d_2(f) = \frac{\Delta^2 V_N(f) - \Delta^2 V_{N/2}(f)}{W(N) - W\left(\frac{N}{2}\right)} + O\left(\frac{1}{N^4}\right)$$

$$d_0(f) = \frac{1}{2}\Delta^2 V_N(\mathrm{f}) - \frac{1}{2} W(N) d_2(f) + O\left(\frac{1}{N^6}\right)$$

$$d_1(f) = V_N(f) - d_0(f) N^2 - \frac{d_2(f)}{n^2} + O\left(\frac{1}{N^4}\right)$$

umumiy formulalar orqali topish mumkin. Bunda:

a) agar

$$V_N(f) = (f - N^2)N^2, \quad f = \lambda_N,$$

deb olinsa yuqorida (7) tenglikda qatnashayotgan koeffitsientlar

$$a_i = d_i(\lambda_N), \qquad i = 0, 1, 2$$

tenglik yordamida aniqlanadi. Xuddi shunday qolgan koeffitsientlarni ham quyidagicha aniqlash mumkin:

b) agar $V_N(f) = (f - N^2)N^2$ va $f = \mu_n$ deb olinsa $b_i = d_i(\mu_N)$

c) agar $V_N(f) = (f - N)N^3$ va $f = \sqrt{\lambda_N}$ deb olinsa, $c_i = d_i(\sqrt{\lambda_N})$;

v) agar $V_N(f) = (f - N)N^3$ va $f = \sqrt{\mu_N}$ deb olinsa, $c_i' = d_i(\sqrt{\mu_N})$;

g) agar $V_N(f) = \left(f - {}^{\pi}/_{2}\right)N^4$ va $f = \alpha_N$ deb olinsa $a_i' = d_i(\alpha_N)$.

Oxirgi satrdagi a_i' ni aniqlashda kichik $N \geq$ n lar uchun (11) da aniqlanadigan α_n qiymatlaridan foydalaniladi. Hisoblashlarda quyidagi belgilashlar kiritilgan:

$$\Delta^2 V_N(f) = V_N(f) - 2V_{N-1}(f) + V_{N-2}(f)$$

$$W(N) = \frac{1}{N^2} - \frac{2}{(N-1)^2} + \frac{1}{(N-2)^2} = \frac{6N^2 - 12N + 4}{N^2(N-1)^2(N-2)^2}$$

$$W\left(\frac{N}{2}\right) = \frac{96N^2 - 384N + 256}{N^2(N-2)^2(N-4)^2}$$

II. Bu bosqichda (7) da qatnashayotgan cheksiz ko‘paytmalarni analitik funksiyalar va chekli ko‘paytmalar orqali ifodalash orqali kichik n<N larda α_n uchun qulay hisoblash formulasi keltirib chiqariladi. Bunda (7) ko‘paytma absolyut yaqinlashuvchi bo‘lganligi uchun uning hadalari ustida guruhlash amallarini bajarish mumkinligi hisobga olinadi [9]. Yuqorida $n >$ N lar uchun topilgan (8), (8’) formulalardan foydalangan holda ba’zi taqribiy almashtirishlardan so‘ng kichik n<N larda o’rinli bo’lgan quyidagi

$$\alpha_n \approx \frac{\pi(c_0' - c_0)}{\mu_n - \lambda_n} \prod_{\substack{k=0 \\ k \neq n}}^{N} \frac{\lambda_k - \lambda_n}{\mu_k - \lambda_n} \prod_{k=0}^{N} \left(\frac{k^2 + \lambda_n'}{k^2 + \lambda_n''} \right) \times$$

$$\times\, exp\left\{ \frac{sh\left(\pi\sqrt{\lambda_n''}\right)\sqrt{\lambda_n''}}{sh\left(\pi\sqrt{\lambda_n'}\right)\sqrt{\lambda_n'}} \right\} + O(N^{-6}) \qquad (11)$$

formulani olish mumkin. Bu yerda $\lambda_n' = 2c_0 - \lambda_n, \quad \lambda_n'' = 2c_0' - \lambda_n$

III. Bu bosqichda (5), (6) tengliklarga asosan h ning qiymati quyidagi formula bo’yicha hisoblanadi.

$$h = -F(0,0) = -\sum_{n=0}^{\infty} \left(\frac{1}{\alpha_n} - \frac{2}{\pi} \right) \qquad (12)$$

Bu yerda (10) formulaga ko’ra (5) cheksiz qatorning umumiy hadlari $1/n^2$ ga ekvivalent bo‘lgani uchun bu qator, shuningdek, (12) qator hadlari ustida guruhlash amallarini bajarish mumkin. Yuqoridagi (12) tenglikning o‘ng tomonidagi qatorni n<N lar va katta n>N lar uchun ikki qismga ajratib yozamiz:

$$h=\sum_{n=0}^{\infty}\left(\frac{1}{\alpha_n}-\frac{2}{\pi}\right)=\sum_{n=0}^{N}\left(\frac{2}{\pi}-\frac{1}{\alpha_n}\right)-\sum_{n=N+1}^{\infty}\left(\frac{1}{\alpha_n}-\frac{2}{\pi}\right)=\mathrm{I}_0-I_1 \qquad (12')$$

a) (12’) qatorda qatnashayotgan α_n o‘rniga uning (10) dagi asimptotik qiymatini qo‘yib, ba’zi elementar shakl almashtirishlardan so‘ng I_1 uchun quyidagi chekli ifodani olish mumkin

$$\mathrm{I}_1=\left(\frac{2}{\pi}\right)^2\left\{\frac{\pi^2a_0'}{6}+\frac{\pi^4a_1'}{90}-\sum_{n=1}^{N}\left(\frac{a_0'}{n^2}+\frac{a_1'}{n^4}-\frac{2\pi(a_0')^2}{(n^2\pi+a_0')^2-A^2}\right)-\right.$$

$$\left.-\frac{(a_0')^2}{A}\left[\frac{\pi\sqrt{\pi}}{\sqrt{(a_0'-A)}}cth\sqrt{\pi(a_0'-A)}-\frac{\pi\sqrt{\pi}}{\sqrt{(a_0'+A)}}cth\sqrt{\pi(a_0'+A)}-\frac{A}{a_1'}\right]\right\}$$

Bu yerda

$$A=\sqrt{(a_0')^2-2a_1'\pi}$$

b) I_0 chekli qatorni hisoblashda esa yuqorida (11) ifoda bilan aniqlangan α_n ning qiymatlaridan foydalaniladi. I_0, I_1 uchun topilgan ifodalarni (12) ga qo’yilganda h uchun ixcham hisiblash formulasi kelib chiqad.

IV. Bu bosqichda (1) tenglamaning

$$\mathrm{u}(0,\mathrm{s}_n)=1, \qquad u'(\mathrm{x},\mathrm{s}_n)=h$$

boshlang’ich shartlarni qanoatlantiruvch yechimini

$$\mathrm{u}(\mathrm{x},\mathrm{s}_n)=\cos(\mathrm{s}_n\mathrm{x})\left(1+\frac{\varphi_1(x)}{s_n^2}+\frac{\varphi_2(x)}{s_n^4}+\cdots\right)+$$

$$+\sin(\mathrm{s}_n x)\left(\frac{\psi_0(x)}{\mathrm{s}_n}+\frac{\psi_1(x)}{s_n^3}+\cdots\right) \qquad (13)$$

ko’rinishda izlash orqali tenglikda qatnashayotgan $\varphi_{i+1}(x)$ va $\psi_i(x)$ lar qiymatini toppish masalasi bilan shug’ullanamiz. Bu yerda

$$\psi_0(x)=h+\frac{1}{2}\int_0^x q(t)dt,$$

qolgan $\varphi_{i+1}(x)$ va $\psi_i(x)$ lar esa $\psi_0(x)$ orqali ifodalanuvchi noma’lum funksiyalardir. Yuqoridagi (13) tenglikning o‘ng tomonidagi qator, agar $q(x)$ funksiya $[0;\pi]$ oraliqda yetarlicha silliq funksiya bo‘lsa, barcha $\mathrm{s}_n>0.5$ lar uchun absolyut yaqinlashuvchi qatordir.

Ma’lumki (1)-(2) masalaning barcha $u(\mathrm{x},\mathrm{s}_k)$ xos fechimlari uchun ushbu

$$\int_0^\pi[\mathrm{u}(\mathrm{t},\mathrm{s}_k)]^2dt=\alpha_k$$

formula o’rinlidir. Integral ostidagi $\mathrm{u}(\mathrm{x},\mathrm{s}_k)$ o’rniga uning (13) dagi ko’rinishini qo’yib $\varphi_i(x),\psi_i(x)$, $(k=1,2,\cdots)$ funksiyalarga nisbatan quyidagi

$$\alpha_k=\frac{\pi}{2}+\frac{\sin(2\pi\mathrm{s}_k)}{4\mathrm{s}_k}+\frac{2}{s_k^2}\int_0^\pi\varphi_1(t)dt+\frac{1}{s_k^4}\int_0^\pi[2\varphi_2(t)+\varphi_1^2(t)]dt+$$

$$+\int_0^\pi \psi_0(t)\frac{\sin(2s_k t)}{s_k}dt + \frac{1}{2s_k^2}\int_0^\pi \left[2\varphi_1(t)-\psi_0^2(t)\right](\cos(2s_k t)-1)dt +$$

$$+\frac{1}{s_k^3}\int_0^\pi \left[\psi_1(t)+\psi_0(\mathrm{t})\varphi_1(t)\right]\sin(2s_k t)\,dt +$$

$$+\frac{1}{2s_k^4}\int_0^\pi \left[2\varphi_2(t)-2\psi_0(\mathrm{t})\psi_1(t)+\varphi_1^2(t)\right](\cos(2s_k t)-1)dt +$$

$$+\frac{1}{s_k^5}\int_0^\pi \left[\psi_2(t)+\psi_0(\mathrm{t})\varphi_2(t)+\psi_1(\mathrm{t})\varphi_1(t)\right]\sin(2s_k t)\,dt + \cdots \qquad (16)$$

tenglamalar seriyasini hosil qilamiz. Bu yerda α_k qiymatlari (11) va (10) formulalar orqali hisoblabnuvchi aniq son. Qulaylik uchun (16) tengliklarni quyidagi

$$\alpha_k-\frac{\pi}{2}-\frac{\sin(2\pi s_k)}{4s_k}=\int_0^\pi \psi_0(t)\frac{\sin(2s_k t)}{s_k}dt +$$

$$+\frac{1}{2s_k^2}\left[\int_0^\pi f_1(t)(\cos(2s_k t)-1)dt + A_1\right]+\frac{1}{s_k^3}\int_0^\pi f_2(t)\sin(2s_k t)\,dt +$$

$$+\frac{1}{2s_k^4}\left[\int_0^\pi f_3(t)(\cos(2s_k t)-1)dt + A_2\right]+\frac{1}{s_k^5}\int_0^\pi f_4(t)\sin(2s_k t)\,dt + \cdots$$

$$\cdots+\frac{1}{2s_k^{2m}}\left[\int_0^\pi f_{2m-1}(t)(\cos(2s_k t)-1)dt + A_m\right]+$$

$$+\frac{1}{s_k^{2m+1}}\int_0^\pi f_{2m}(t)\sin(2s_k t)\,dt + \cdots \qquad (16')$$

$$m=1,2,\cdots.\ f_{-1}=A_0=0, \qquad f_0=\psi_0(t)$$

ko'rinishiga keltirish orqali $\psi_0(x)$, $f_m(x)$ va A_m, noma'lumlarga nisbatan chiziqli tenglamalar seriyasini hosil qilamiz. Keyingi V bosqichda keltiriladigan usul yordamida topilgan $\psi_0(x)$ va $f_m(x)$ lar asosida noma'lum $\varphi_m(x), \psi_m(x)$ funksiyalga nisbatan quyidagi

$$\varphi_1(x)=\frac{1}{2}\left[f_1(x)+\psi_0^2(x)\right],\ \ \psi_1(x)=\left[f_2(x)-\psi_0(\mathrm{x})\varphi_1(x)\right],$$

$$\varphi_2(x)=\frac{1}{2}\left[f_3(x)+2\psi_0(\mathrm{x})\varphi_1(x)-\varphi_1^2(x)\right],$$

$$\psi_2(x)=\left[f_4(x)-\psi_0(\mathrm{x})\varphi_2(x)-\psi_1(\mathrm{x})\varphi_1(x)\right],\cdots$$

formulalarni olish mumkin. O'z navbatida bu tengliklardan $y(x, s_n)$ xos funksiyalarni tiklashda foydalaniladi.

V. Ushbu bosqichda noma'lum $\psi_0(x)$ va $f_i(x)$ funksiyalarni aniqlash bo'yicha chiziqli tenglamalar sistemasini qurish masalasi muhokama qilinadi.

Faraz qilaylik (16′) tenglikning o'ng tomonida $2m+1$ ta had qatnashsin. Yuqorida xatolikga nisbatan qo'yilgan talabga ko'ra N dan kichik s_k larda $(16')$ tenglik $O(N^{-6})$ aniqlikni berishi uchun

$$O(s_k^{-2m-1}) \approx O(N^{-6})$$

ekvivalentlik o'rinli bolishi kerak. Bundan $(16')$ tenglikda yetarli aniqlik ta'minlanishi uchun m ga nisbatan quyidagi

$$m \geq \left[\frac{3lnN}{\ln(s_k)} - 1\right] \tag{17}$$

talab kelib chiqadi. Bu o'rinda albatta $\ln(s_k) > 0$ bolishi hisobga olinadi.

Yuqoridagi mulohazaga ko'ra $(16')$ tenglika $2m + 1$ ta integral qatnashadi. Agar har bir integral $[0, \pi]$ oraliqda bo'linishlar soni N_1 ga teng bo'lgan integral yig'indi bilan almashtirilsa $(16')$ ning har bir tenglamasida $(2m + 1)N_1 + m$ tadan noma'lum qatnashadi. Shundan kelib chiqib quyidagi mulohazalarga ko'ra $(2m + 1)N_1 + m$ noma'lumli $(2m + 1)N_1 + m$ tenglamalar sistemasini quramiz. Mulohazalarga ko'ra indekslari $k_1 < k < k_2$ oraliqda bo'lgan tenglamalar seriyasini qaraymiz.

Ma'lumki, (1)-(2) masalaning xos qiymatlari $0 < s_1 < s_2 < \cdots$ tartibda o'sib boradi. Faraz qilaylik tuzilayotgan tenglamalar sistemasiga $(16')$ ning k_1 indeksdan boshlab keyingi $(2m + 1)N_1 + m$ ta tenglamasini tanlab olaylik. Noma'lumlar sonini mumkin qadar kamaytirish maqsadida (17) tenglikka ko'ra k_1 ni (ya'ni etarli katta s_{k_1} qatnashayotgan tenglamadan boshlab) mumkin qadar katta olish kerak. Ayni paytda (9) va (10) tenliklarga ko'ra yetarli katta $k > N^5$ indekslardan boshlab $(16')$ da qatnashayotgan tengliklar o'zaro kollinrar tengliklarga aylanib boshlaydi. Shu mulohazalarga ko'ra tuzilayotgan tenglamalar sistemasiga o'zaro kollinear bo'lgan tenglamalar qatnashmasligi uchun tanlangan so'ngi tenglamaning indeksi $k_2 << N^5$ bo'ishi kerak. Shu mulohazalarga ko'ra

$$\min(k_2) - \max(k_1) = (2m + 1)N_1 + m$$

shartni qanoatlantiradigan intervaldagi tenglamalar tanlanishi kerak.

VII. Ushbu bosqichda (2) chegaraviy shartdagi H ning qiymati aniqlanadi. Ma'lumki, 1-lemmaga asosan aniqlangan (9) tenglikdagi c_0 koeffisient uchun quydagi

$$c_0 = \frac{1}{\pi}\left[h + H + \frac{1}{2}\int_0^{\pi} q(t)dt\right] = \frac{1}{\pi}\left[H + \psi_0(\pi)\right]$$

tenglik o'rinlidir [1]. Bundan H ning qiymatini aniqlash mumkin bo'lgan quyidagi

$$H = \pi c_0 - \psi_0(\pi)$$

tenglika ega bo'lamiz. Bu yerda c_0, $\psi_0(\pi)$ lar qiymatlari yuqoridagi bandlarda hisoblab topilgan.

Xulosa. Shturm-Liuvill operatori parametrini tiklash masalasini yechish uchun taklif qilingan usul yordamida samarali hisoblash algoritmi ishlab chiqilgan va kompyuter dasturi tuzilgan. Hisoblash tajribasi h, H larning har xil qiymatlari va har xil ko'rinishdagi $q(x)$ manbaa funksiyalari uchun topilgan xos qiymatlar seriyasida sinovdan o'tkazildi. Haqiqiy ma'lumotlar va tavsiya qilingan usulda topilgan natijalar solishtirildi, xatoliklar baholandi. Kuzatish natigalari ma'lumotlar yetarli bo'lmagan sharoitda teskari masalani yechish boʻyicha maqolada tavsiya qilingan usulning yuqori aniqliligi, ixchamliligi va har xil masalalarga moslanuvchanligini ko’rsatdi.

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